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PROBLEMS OF HYDRODYNAMICS OF FLOWING FILMS

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A brief analysis is given of the works on the hydrodynamics of a flowing film. The basic model representations of a flow are considered. Possibilities of using statistical-mean wave characteristics of a film flow for further development and refinements of semiempirical theories of transfer in flowing films and design procedures for film technological apparatuses and power cooling systems are pointed out.

Expanding the use of spraying films in technological equipment and cooling systems and developing and promoting the methods to intensify transfer processes in them are limited mostly by insufficient development of the theory and, especially, by a lack of complex experimental data. This primarily concerns new promising trends associated with the study and optimal use of regularities of wave phenomena in gravitational films and with special cases of transfer in them while flowing on glass enamel, polymer, profiled, vibrating, and other surfaces.

The one- or two-dimensional flow of a film which can be produced under laboratory conditions with very low water concentrations are considered in the well-known theoretical works [1-3]. Under ordinary conditions, a random three-dimensional wave regime of flow occurs and so this is of primary practical importance. Since a mathematical description of the three-dimensional flow of a liquid presents great difficulties, the basic method of a quantitative study is an experimental investigation. However, conducting such an experiment under conditions of wave, most commonly nonisothermal, flow of a thin liquid layer, whose oscillating thickness rarely exceeds 1.5-2.0 mm, and applying the methods and procedure of measuring the basic parameters are extremely difficult and tedious and often call for the development of special sensors, instruments, and devices [4].

Only a few very detailed works focuse attention on wave phenomena in a film. Ignoring or insufficiently accounting for the wave character of a flow leads to considerable distortions of the real mechanism of transfer in the film and to large inaccuracies in the results of calculation, which can be an order of magnitude different from the most accurate experimental data obtained recently, the structure of surface waves having a determining influence on the mechanism of the processes of momentum, heat, and mass transfer in the film.

Wave Flow of a Gravitational Film. Waves emerge even with very low water concentrations on a free film surface; they were first visually studied by Hopf [5]. Wave formation considerably transforms the processes of transfer in a thin liquid layer, which was taken into account in the first stage of studying film processes by introducing experimental correction factors into the calculation equations.

The first fundamental theoretical investigation of the wave flow of a liquid was performed by P. L. Kapitsa, who presented an equation of the free film surface in the form

$$y = \bar{\delta}[1 + f_w(x, \tau)], \tag{1}$$

where f_w is an arbitrary function. Under the assumption that at small Reynolds numbers the free film surface can be described by a sinusoidal function, the waves have a profile that does not change and they move with a constant phase velocity; under a number of other simplifications, the Navier-Stokes equations, the flow continuity equations, and Eq. (1) were simultaneously solved for the case of flow over a vertical surface.

The theory of laminar-wave flow of a liquid film in the linear approximation was subsequently refined [6] and developed by domestic and foreign scientists. A survey and critical analysis of their works is given in [7-9]. A more general theory in a nonlinear statement was recently developed for both the free flow of a film [2] and the flow of films

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interacting with a gas flow [10, 11]. Subsequent works substantiated the possibility of existence of noncapillary waves on the film surface [12], with emphasis on studying the influence of the physicochemical properties of the liquid on the wave characteristics [3].

Theoretical works deal, as a rule, with two-dimensional regular wave disturbances in flowing films, and only individual works [9] deal with three-dimensional ones. In this case there are two approaches: in the first approach the function of the outer wave surface is prescribed in the form of a sinusoid or some other, more intricate trigonometric function; in the second approach use is made of the theory of small progressive disturbance waves, i.e., progressive Tollmien-Schlichting waves are superposed on a laminar film [9] and possibilities of exciting or damping these superposed disturbance waves are investigated. Since it is difficult to analytically describe the free film surface, the range of accuracy and the field of application of the obtained results depend, in the first approach, on how adequately the chosen function of the outer film surface describes the real wave structure under these conditions. In the second approach one usually circumvents describing the free outer film surface by a certain analytical function and, since complete solutions cannot necessarily be found, the problem is solved by a numerical method.

Analyzing the solutions of the differential Orr-Sommerfeld disturbance equation enabled us to obtain [9] the result that in the case when the dimensionless Tollmien-Schlichting wave number $\alpha_w = 2\pi\delta/\lambda_w \to \infty$, i.e., at very small wavelengths λ_w, the wave flow of the film is always stable. A spraying film is always unstable under long-wave disturbances; convective instability is characteristic of it in this case, with the possibility of laminar smooth flow not excluded [13]. The equation for the instability of a flowing film becomes [9]

$$\frac{8}{5} > \frac{\alpha_w^2 \sigma}{\delta u^2 \rho} + \frac{g\delta \cos\varphi}{u^2}, \tag{2}$$

and for the other boundary case $\alpha_w \to 0$, i.e., the wavelength $\lambda_w \to \infty$, the first term disappears and after we have expressed the second term as the film Reynolds number we have finally the instability condition

$$\mathrm{Re} > \frac{10}{3} \operatorname{ctg} \varphi. \tag{3}$$

Expression (2) shows that the surface tension σ of the liquid has a stabilizing influence on the flow since condition (3) can also be obtained from (2) as $\sigma \to 0$. Therefore, it is natural that the appearance of a surface tension gradient caused by the emergence of a temperature gradient on the film surface, transverse to the direction of flow, or of a PAV concentration gradient leads to a change in the flow stability (the thermocapillary effect, the Marangoni effect) [3, 14, 15] and a deformed velocity profile. The appearance of the above longitudinal gradients in the film leads to a change in the physical properties of the liquid and to a change in the relation between inertial and viscous friction forces along the flow. Mass transfer on the outer surface also affects the stability of the flow: thus, the film stability increases with film condensation of a vapor and decreases with evaporation.

From condition (3) it can also be concluded that for a vertical spraying film there is no critical Reynolds wave number $\mathrm{Re}_{cr.w}$, below which there would be no excited waves. As soon as the disturbance waves cease to be perpendicular to the basic direction of flow (flowing down the wall with the slope φ), according to the theory, there is such a value: $\mathrm{Re}_{cr.w} = 10/3 \cot\varphi$. In the case of a vertical wall, these waves are difficult to record since the amplitudes are too small and the wavelengths are too large.

According to the linear theory of wave flow [9], a sudden rise of the excitation factors in the vertical spraying film begins with $\mathrm{Re} = 0.864\ K_f^{0.125}$, which can be conveniently considered as a quasicritical Reynolds wave number. Probably, it is precisely this moment that has been identified in experimental and some simplified theoretical works with the emergence of the first regular wave disturbances in the vertical film.

The emergence of waves on the surface of a vertical gravitational film is observed experimentally even at Re = 12-30; various dependences are given for calculating the value of $\mathrm{Re}_{cr.w}$: $\mathrm{Re}_{cr.w} = 2.43\ K_f^{1/11}$ [16], $1.2\ K_f^{0.1}$ [17], $1.164\ K_f^{0.125}$ [18], etc., and the experimental results disagree by as much as 500%, which can easily be explained by different experimental conditions and differences in accuracy. Wave formation is observed not on the entire liquid surface but at some distance from the sprayer [19]. The first emerging wave has a regular two-dimensional character; their amplitude increases linearly with increasing Reynolds number, and the wave front is perpendicular to the direction of flow and remains almost undistorted for the entire time. "Long" gravitational waves whose length decreases as the flow velocity increases, are characteristic of this regime of flow, which is sometimes referred to as the first laminar wave regime. But as the flow rate of the liquid becomes 4-5 times as large, the instability and three-dimensionality of the wave regime emerge. The emergence of this, the so-called second laminar, regime is related not only to the bend in the wave front and the emergence of "oblique" waves but also to their shortening. The parameters of wave formation depend mainly on capillary forces [2-4, 20]. A further increase in the water concentration leads to development of "shock" or "overflow" waves as well as to superposition of smaller capillary waves on them [1, 3, 4, 14, 20] and to enhancement of transverse mixing in the film. The basic regimes of wave flow, the profiles of the outer film surface, and the basic calculated dependences are given in greater detail in [3, 4, 10, 14, 17, 20-22].

Theoretical works on a wave flow of a liquid film in a linear approximation describe, as a rule, the first laminar wave regime. A nonlinear theory of wave flows gives results, confirmed fairly well by experiments up to quite large Reynolds numbers ($Re \le 200$-600), which can be used to design technological heat- and mass-transfer apparatuses operating with very low water concentrations. Industrial apparatuses and cooling systems usually operate with much larger flow rates of the liquid.

Developed Turbulent Flow of Gravitational Films. A transition to developed turbulent flow occurs in the region $1200 \le Re \le 2400$. For ordinary spraying surfaces of the apparatus and treatable liquids, not weakened by PAV, the critical Reynolds number Re_{cr}, corresponding to this transition, is taken equal to 1600 [3, 4, 10, 17].

In an effort to theoretically describe a film flow in the developed turbulent regime one commonly assumes a velocity profile in the flow, which is similar to the universal Prandtl-Karman velocity profile. It is more difficult to describe the development of turbulence in a film than in a continuous confined liquid flow since the size of vortices that form is limited to the thickness of the film undergoing weak-stationary three-dimensional wave oscillations.

In the well-known two-layer model of the turbulent flow of a film [1, 23] with a smooth outer surface one considers a laminar sublayer, for which $y \le \delta_{lam}$:

$$u = \frac{g}{\nu}\delta y, \tag{4}$$

and a turbulent layer, for which $y \ge \delta_{lam}$:

$$u = u_{lam} + \frac{1}{\varkappa}(\rho g)^{0,5}\left\{2(\delta - y)^{0,5} - 2(\delta - \delta_{lam}) + \delta^{0,5}\ln\frac{[\delta^{0,5} - (\delta - y)^{0,5}][\delta^{0,5} + (\delta - \delta_{lam})^{0,5}]}{[\delta^{0,5} + (\delta - y)^{0,5}][\delta^{0,5} - (\delta - \delta_{lam})^{0,5}]}\right\}. \tag{5}$$

Here $\kappa = 0.4$ is the turbulence constant [23].

In the absence of friction on the outer surface of the film ($\tau_{out} = 0$):

$$Re = 4\left(\frac{g\delta^3}{\nu^2}\right)^{0,5}\left\{11{,}6 + 2{,}5\left[\ln\frac{1}{11{,}6}\left(\frac{g\delta^3}{\nu^2}\right)^{0,5} - 1\right]\right\}. \tag{6}$$

This solution is in fairly good agreement with experimental data, although the real velocity profile in the turbulent spraying film is different, and the outer film surface is covered with variously structured waves.

In the analytical Dukler-Bergelin solution a three-layer model of a flow is used: a laminar viscous sublayer, a transition (buffer) region, and a region of developed turbulence near the outer surface, and the velocity distribution in the layers is described according to the equations [24]

$$0 \leqslant y^+ \leqslant 5 \qquad u^+ = y^+, \tag{7}$$

$$5 < y^+ \leqslant 30 \qquad u^+ = -3{,}05 + 5\ln y^+, \tag{8}$$

$$30 < y^+ \leqslant \delta^+ \qquad u^+ = 5{,}5 + 2{,}5\ln y^+, \tag{9}$$

where the dimensionless distance y^+ from the wall and velocity u^+ are equal to:

$$y^+ = u^* \frac{y}{\nu}, \quad u^+ = \frac{u}{u^*}.$$

Here $u^* = (\tau_{wa}\rho^{-1})^{1/2}$ is the dynamic viscosity; $\delta^+ = \bar{\delta} u^* \nu^{-1}$ is the dimensionless film thickness.

For the case of turbulent flow we obtain

$$\mathrm{Re} = 4\delta^+ (3 + 2{,}5 \ln \delta^+) - 256. \tag{10}$$

by using the expression for the Reynolds film criterion and Eqs. (7)-(9). If there are no tangential stresses on the free film surface

$$\delta^+ = \bar{\delta}^{1,5} \nu^{-1} \sqrt{g \sin \varphi}. \tag{11}$$

The experimental check of this method using the universal Prandtl-Karman velocity profile yielded satisfactory results for the laminar sublayer and the turbulent region where it can be described by one equation [25]:

$$y^+ = u^+ + 0{,}1118 (e^{0{,}281 u^+} - 1), \tag{12}$$

and somewhat worse results in the buffer zone. However, the implicit form of the function (12) assumes numerical or graphical methods of solution [25].

Since the spraying film has the free surface with developed wave formation the mechanism of pulsation phenomena within the liquid flow is different from that in flow through pipes. With $\mathrm{Re} > \mathrm{Re}_{cr}$ the turbulent pulsations within the flow, reaching the outer surface, excite it and strive to disrupt it, which is prevented by the surface tension forces. The pulsation energy is partly spent on bending the film surface and partly dissipated as heat. With developed turbulence in the film its free surface is deformed by wave oscillations of a different nature and structure caused by both the instability under the external disturbances and the joint action of turbulent vortices and surface tension forces. Large low-frequency shock waves or outflows are covered with a net of high-frequency oscillations [3, 10, 17, 20]. This wave structure in the developed turbulent flow of a film has been recorded by many investigators with the aid of high-speed filming and photographing and accurate measurements.

The semiempirical theory of turbulent transfer was further developed in [26], in which the dynamic velocity is expressed in terms of the energy dissipated by turbulent pulsations in the wall layer of the liquid. Based on an analysis of power consumption in variously designed thin-layer apparatuses, expressions for calculating the dynamic viscosity and simplified calculated dependences of heat transfer were found. Subsequently, this method was also used by G. Gimbutis [13] under various conditions of momentum and heat transfer in a gravitational film.

For the practical use of Eqs. (10)-(12) it is necessary to determine the friction rate u^* and to know the tangential stress near the wall τ_{wa}. This method also does not take into account the effect of wave formation on the outer film surface on the character of the stream flow; this situation calls for introducing empirical corrections and refinements [27], and such an approach is justified only for fairly thin films under certain conditions of their flow.

In other models for obtaining an equilibrium value of the mean film thickness in turbulent flow the power velocity distribution is used [3], which corresponds more closely to observations.

The models considered have basically two common flaws [10, 14]: discontinuities of the velocity derivative at the points of layer (zone) cross-linking and the impossibility of accurately calculating the velocity on the free surface. The model of the turbulent flow of a smooth film developed in the works of N. N. Kulov, V. A. Malyusov, and others [10, 14] is more universal. This model takes account of the tangential stress distribution and the influence of the molecular viscosity in the entire flow and makes it possible to calculate both the profile of the averaged velocity in the film and the velocity on its free surface. Measurements of the profiles of the time-averaged local velocities in a water film using a laser Doppler anemometer have shown [10] that in developed turbulent flow the character of the distribution of pulsation velocities substantially differs from those postulated in the theories [14].

Taking into account the random nature of the wave process, the most correct approach to studying it is a probabilistic determination of the basic wave characteristics using the spectral density of the power of the wave process, which enables one to obtain information on the contribution of all frequencies and to determine the most probable harmonic [3, 20], with the basic amplitude-frequency characteristics of the wave flow being calculated with the aid of random function theory. These statistical-mean characteristics of the wave flow of spraying films can be used to describe the mechanism of transfer, to refine model representations, and to develop new methods and devices for intensifying heat and mass transfer in film apparatuses and cooling systems [20, 28, 29].

NOTATION

g, free fall acceleration, m/sec^2; u, velocity, m/sec; x, coordinate axis in the direction of flow, m; y, coordinate axis normal to the direction of flow, m; α_w, wave number; Γ_v, volume concentration of water, m^2/sec; δ, local film thickness, m; λ_w, wavelength, m; μ, dynamic viscosity of a liquid, Pa·sec; ν, kinematic viscosity of a liquid, m^2/sec; ρ, density, kg/m^3; σ, surface tension of a liquid, N/m; τ, time, sec; τ_{wa}, tangential stress, N/m^2; φ, slope of the sprayed wall, deg; $K_f = \rho\sigma^3/g\mu^4$, film number (Kapitsa criterion); $Re = 4\delta u/\nu = 4\Gamma_v/\nu$, Reynolds criterion for a film flow. Subscripts: w, wave; cr, critical; lam, laminar; out, outer; f, film; ω, wall; 0^+, dimensionless, $\bar{0}$, mean (averaged).

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